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A new buoyancy model replacing the standard pseudo-density difference for internal natural convection in gases

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Abstract

Natural convection in enclosed spaces has been analyzed and numerically evaluated using a direct buoyancy model which replaces the pseudo-density difference which has been heretofore standard in all previous studies of internal natural convection of gases. It was demonstrated conclusively that the pseudo-density-difference model is totally irrelevant to the physical processes which create the buoyancy. When properly accounted, the presence of the pseudo-density difference does no harm, but it provides a source of confusion for the source of the buoyant forces which create motion. This conclusion was drawn from numerical solutions of three-dimensional natural convection in an oven-like cavity. These solutions were irrefutably supported by experimental data. It was also demonstrated that accounting for the naturally occurring pressure variations within the enclosed space had a negligible effect upon the surface heat transfer coefficients. The commonly used Boussinesq equation of state was found to provide accurate surface heat transfer results provided that the density, thermal conductivity, viscosity, and coefficient of thermal expansion are evaluated at a temperature that is the average of the temperatures of the surfaces which bound the enclosed space.

1. Introduction

The history of analytical solutions to natural convection problems had its origin with the pioneering paper of Schmidt and Beckmann [1] that dealt with the boundary layer on a vertical heated plate. That paper marked the advent of the use of the Boussinesq model for calculating the density differences that occur across the boundary layer. The occurrence of the density difference in the Schmidt and Beckmann problem is based on the logical representation of the vertical pressure gradient in the boundary layer by the vertical pressure gradient in the quiescent environment outside the boundary layer. Inasmuch as the computational tools available to Schmidt and Beckmann were quite limited, it was necessary to transform the naturally occurring density difference to a corresponding temperature difference, and that transformation was accomplished by the Boussinesq equation of state. Subsequent work on natural convection boundary layers involved naturally occurring density differences which were always transformed to temperature differences in the aforementioned manner.

Problems of internal natural convection first attracted attention several decades after the Schmidt and Beckmann work. By that time, workers in the field had become accustomed to the presence of a naturally occurring density difference and continued to employ the Boussinesq approach; however, a forthright appraisal of natural convection in enclosed spaces does not reveal a naturally occurring density difference. The early practice of employing such a density difference appears to have

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Nomenclature

g	gravitational acceleration (m/s^2)
$\bar{h}_{BOT}$	surface-averaged convective heat transfer coefficient on the bottom surface of the thermal load (W/m^2 K)
$\bar{h}_{FRONT}$	surface-averaged convective heat transfer coefficient on the front surface of the thermal load (W/m^2 K)
$\bar{h}_{REAR}$	surface-averaged convective heat transfer coefficient on the rear surface of the thermal load (W/m^2 K)
$\bar{h}_{SIDE}$	surface-averaged convective heat transfer coefficient on the left and right surfaces of the thermal load (W/m^2 K)
$\bar{\bar{h}}$	all-surface-averaged convective heat transfer coefficient on the thermal load (W/m^2 K)
P	local pressure (N/m^2)
P_0	hydrostatic pressure (N/m^2)
P'	reduced pressure ($P - P_0$), (N/m^2)
T	temperature (K)
T_0	reference temperature (K)
y	vertical coordinate (m)
Greek symbols	
β	coefficient of thermal expansion (K^{-1})
ρ	density (kg/m^3)
ρ_0	hydrostatic density (kg/m^3)

influenced all the computational investigations of internal natural convection. Indeed, this practice continues to the present day, as witnessed by the numerous "benchmarking" solutions that have been published over the last several decades [2–9].

For internal natural convection, the pseudo-density differences were created by algebraic manipulation of the vertical pressure gradient. This pseudo-density difference implies a physical mechanism for buoyancy that is different from that which actually occurs in internal natural convection. Inasmuch as the solutions of natural convection problems based on the Boussinesq model do not require a knowledge of the pressure field within the enclosed space, the aforementioned manipulation could be utilized without complication.

Two recent papers [10,11] dealing with internal natural convection appear, at first glance, to have made a major departure from the previously standard Boussinesq approach based on the aforementioned pseudo-density difference. Those studies continue the practice of employing a pseudo density-difference, but use the ideal gas law instead of the Boussinesq equation of state. This treatment contains the logical inconsistency associated with the pseudo-density term although it is an improvement over the past in that it uses a more appropriate equation of state.

In evaluating the ideal gas law, the pressure field within the enclosed space is necessary. In this regard, it is necessary to pay very careful attention to the formulation of the pressure boundary conditions. From an examination of the analyses presented in [10,11], it was found that there was no mention made of a pressure boundary condition. Furthermore, in the creation of the pseudo-density term, the fictive subtractive density contribution was treated in dissimilar ways in the two references being discussed. In [10], this subtractive term was a constant, while in [11], the subtractive term was an exponentially varying function of the vertical coordinate. The different approaches to the subtractive term in the two references is the result of a different assumption about the nature of the "hydrostatic" density variation. Since neither of these subtractive densities is true to reality, it is not remarkable that different models were used to describe them. In fact, the notation of a hydrostatic state is, itself, a mental construct.

Surprisingly, it may be noted that [11] did not take note of the existence of [10] which had been published two years earlier. This is understandable because the issue of the treatment of the buoyancy term via the ideal gas law was a side issue in [10] and was mentioned only in a single sentence imbedded in the interior of the paper.

Although not appearing in the published literature, the treatment of buoyancy in the well-known FLUENT CFD software also includes a pseudo-density difference [12]. In order to restore the FLUENT natural convection model to its unadulterated state, the present authors have eliminated the pseudo-density difference.

In the present investigation, which encompassed *three-dimensional* natural convection in an enclosed space having a centered body, the body force was evaluated by the ideal gas law without the complication of creating a pseudo-density difference to construct a buoyancy force. Therefore, the ambiguities and logical inconsistencies which were cited in the foregoing paragraphs were not encountered. Furthermore, careful attention was paid to the proper specification of a pressure boundary condition in recognition of the fact that the ideal gas law requires the pressure field for its proper evaluation. For further perspective, two versions of the ideal gas law were utilized. In one, the pressure was permitted to vary naturally throughout the enclosed space, whereas in the other, the density was evaluated at a constant pressure. Additional perspective was obtained by employing the full ideal gas formulation while retaining the pseudo-density term. Finally, the widely used Boussinesq equation of state was also examined from the standpoint of comparison of the solutions obtained there from the solutions obtained from the physically correct model.

In addition to considering three-dimensional issues, the present work took account of various heating modes at the enclosure wall including the presence of a discrete patch (due to a rod-like heater) and continuous heating away from the discrete patch. The centered body modeled the presence of a thermal load such as that which might be encountered either in food or materials processes.

To provide the ultimate verification of the present computational work, the heat transfer results predicted by the use of the full ideal gas formulation were compared with experimental data collected by the authors [13].

2. Treatment of the body force

In the momentum equation for the vertical direction (y being positive upwards), the naturally occurring pressure and body force terms are

$$-\frac{\partial P}{\partial y}-\rho g. \tag{1}$$

In the approach adopted by the authors, these terms were treated as they appear in Eq. (1). On the other hand, to the best knowledge of the authors, all prior investigators of internal natural convection have employed a pseudo-density difference whose derivation begins with the introduction of a fictive hydrostatic state defined by

$$-\frac{\partial P_0}{\partial y}=\rho_0 g, \tag{2}$$

where P_0 and ρ_0 respectively denote the hydrostatic pressure and density. For the most part, the density ρ_0 has been taken to be a constant, but in [11], it was evaluated via the ideal gas law at a uniform temperature. When the hydrostatic state is introduced into Eq. (1), the pseudo density-difference appears

$$-\frac{\partial(P-P_0)}{\partial y}-(\rho-\rho_0)g. \tag{3}$$

The next step in the derivation of the heretofore standard buoyancy model is to define a reduced pressure

$$P'=P-P_0, \tag{4}$$

which leads to

$$-\frac{\partial P'}{\partial y}-(\rho-\rho_0)g. \tag{5}$$

Eq. (5), with or without the prime ($'$), is the generally encountered representation of the buoyancy term for internal flows.

The shift in the pressure variable as indicated in Eq. (4) is of little relevance when the Boussinesq equation of state is used to evaluate $(\rho-\rho_0)$. This is because $(\rho-\rho_0)$ is replaced by a $\beta\rho(T_0-T)$, where ρ is an externally prescribed quantity and is not calculated by the computer program. Indeed, once the Boussinesq approximation has been made, all of the densities which appear in the conservation laws are evaluated as the same externally prescribed constant. If the ideal gas law were to be used to evaluate the density, then the shift in the pressure variable as indicated in Eq. (4) must be carefully accounted for when the pressure is introduced into the ideal gas law.

It is clear from the foregoing that the pseudo-density difference introduced in Eq. (3) is a fabricated construct which has little to do with the physical processes which create buoyancy within an enclosed space. Its presence does no harm provided that it is properly accounted. It is the view of the present authors that the fabrication of the pseudo-density difference is not only irrelevant to the physical understanding of the buoyancy process as it occurs in enclosed spaces, but also may lead to numerical errors if it is not properly accounted. It is the further belief of the authors that the logical approach to modeling buoyancy in enclosed spaces is to make use of the naturally occurring terms that appear in Eq. (1).

3. Problem definition

The physical problem chosen to demonstrate the use of the direct formulation of the buoyancy term, Eq. (1), was selected because of the availability of experimental data to test the accuracy of the computational predictions. The geometry of the internal space consisted of a three-dimensional, rectangular enclosure within which there was a center body. This situation was intended as a model for the heating of a thermal load situated in an oven. The oven walls were maintained at a uniform temperature which exceeded the uniform temperature assigned to the thermal load (450 and 300 K respectively). In addition to the heating of the load due to the elevated temperatures of the oven walls, further heating was provided by a linear heat source situated on the floor of the oven. It is believed that the complexity of the selected problem provided a demanding test of the buoyancy model that has been described in the foregoing section of this paper.

Four different cases were investigated for the aforementioned geometry and thermal boundary conditions. These included:

- Ideal gas model used to evaluate the density variation throughout the oven cavity. This model did not include the presence of the pseudo-density term, and, therefore, represents the *natural* approach to the buoyancy evaluation. The density variation was incorporated into all terms in the conservation equations in which the density appears.
- Ideal gas model used to evaluate the density variation throughout the oven cavity. This model included the presence of the pseudo-density term. For this case, in common with the practice adopted for the preceding case, the density variation was incorporated into all terms in the conservation equations in which the density appears.
- Modified ideal gas model in which the pressure variations throughout the oven were omitted in the evaluation of the density (*isobaric* ideal gas model). This model also included the presence of the pseudo-density difference. As in the prior cases, the density variation was incorporated into all of the relevant terms of the conservation equations.
- Boussinesq equation of state was used for the evaluation of the buoyancy term. The density which appears in all the other terms of the conservation equations was evaluated at a constant corresponding to a pressure of one atmosphere and a reference temperature which was the average of the respective temperatures of the oven walls and the thermal load.

In all of the cases just described, the other thermophysical properties k and μ, respectively the thermal conductivity and the viscosity, were evaluated at the reference temperature.

The software selected for the numerical simulations of the convective flow and heat transfer was the FLUENT 6.0 program. In the default mode for internal natural convection flows, the program provides an internally calculated value of the quantity ρ_0 which appears in Eq. (5). The calculation of ρ_0 was performed by averaging the local densities in each of the control volumes that constituted the solution domain. In order to eliminate the pseudo-density difference, the present authors manually set $\rho_0 = 0$.

The solution domain was subdivided into approximately 850,000 control volumes in order to properly resolve the small-scale features of the fluid flow and temperature fields. Accuracy was enhanced by the use of a double-precision solver. The use of this solver was motivated by the fact that the discretized equations involved the differencing of numbers of very similar magnitude such as those that occur in the velocity field.

The Rayleigh number which characterized the solutions was 6×10^5 based on the vertical height of the thermal load. Had the Rayleigh number been based on the vertical height of the oven cavity, its value would have been 6×10^8.

Two complementary numerical techniques were employed to stabilize the iterative mode of solution. One of these involved the fictive use of reduced gravity to artificially diminish the Rayleigh number during the early stages of the calculations. As the calculations proceeded, the gravity value was successively increased to its proper value. The second approach made use of the under-relaxation factors for the various transport equations. It was found highly advantageous to use minimal values of the under-relaxation factor for the momentum equations. These values were in the range of 0.01–0.05. For the other transport equations, relaxation factors between 0.1 and 0.3 were typical.

Numerical experiments were performed to seek the most appropriate mode of treating the convection terms. Investigated approaches included both first-order upwinding and second-order upwinding. Since the latter requires greater computational time to implement, the former was used during the initial iterations for a given case, and the latter was brought online when the converged solution was approached. For the pressure–velocity coupling, the SIMPLE algorithm was used.

The numerical simulations were carried out under the assumption of laminar flow and heat transfer. The choice of the laminar model was motivated by the Rayleigh number range of the analyzed physical system, the largest value of which was 6×10^5.

All of the numerical solutions were obtained by use of an IBM SP supercomputer. About 50,000 iterations were required to converge the solutions for the respective cases. Convergence was identified by monitoring the surface heat transfer rates at the respective surfaces of the thermal load. The attainment of convergence was identified by the constancy of each of the surface heat flows to three significant figures as the number of iterations was systematically increased.

4. Results and discussion

4.1. Surface heat transfer coefficients

From the standpoint of engineering practice, the quantities of greatest interest are the heat transfer coefficients at the various surfaces of the thermally active object. In the present instance, the thermally active object is the load that is centrally situated in the oven cavity. The load is a six-sided block. In addition to the heat transfer coefficients $\bar{h}$ that are specific to each of the individual surfaces, the all-surface-average heat transfer coefficient $\bar{\bar{h}}$ is also tabulated. Note that, due to symmetry, a single value of $\bar{h}$ is applicable for two of the side surfaces. The heat transfer coefficients for the respective sides are listed in Table 1 for each of the four models of the buoyancy term that were described earlier.

By comparing the four columns of results exhibited in the table, it is seen that there is remarkable agreement among the heat transfer coefficients provided by the various models. Of particular interest are the second and third columns. The virtually perfect agreement evidenced in these columns verifies the physical expectation that the pseudo-density difference does no harm when it is properly accounted.

Examination of the fourth column of the table reveals the effect of not accounting for the pressure variations throughout the enclosed space. The impact of this neglect is seen to be of the order of 1–3%, depending on the side of the load being considered. The results based on the Boussinesq model are listed in the fifth column of the table. The apparent remarkable accuracy of these results must be viewed in light of the manner in which the density, thermal conductivity, and viscosity were evaluated. These quantities were determined at a reference temperature that was the average of the temperatures of the oven walls and the surfaces of the thermal load. The use of a reference temperature of this prescription is the most commonly specified recipe for property evaluation. The results of the Boussinesq model displayed here strongly reinforce the use of this reference temperature.

It is relevant to compare the calculated results displayed in Table 1 with experimental data reported in [13]. In that reference, information is provided for the combined effects of natural convection and thermal radiation. For one of the cases for which experiments were performed, the radiative effects were minimized by employing surfaces of very low emissivity (0.03). After correcting for the small radiation contribution for that case, the all-surface-average convective heat transfer coefficient was found to be 5.97 W/m^2 K. This experimental result is in astonishingly good agreement ($\approx$1%) with the $\bar{\bar{h}}$ value presented in Table 1 for the ideal gas model. Even for the $\bar{\bar{h}}$ which corresponds to the Boussinesq model, the deviation is just under 3%.

4.2. Temperature field comparisons

A further evaluation of the effect of the different models of buoyancy is provided by an examination of the temperature fields within the enclosure. Inasmuch as the problem is three-dimensional, a full presentation of the temperature field would require a large number of figures. For the present purposes, it is sufficient to show the temperature field in a symmetry plane that bisects the thermal load and the oven cavity. Contour diagrams showing isotherms are presented in Figs. 1–4, respectively for the four investigated cases.

Inspection of the figures reveals virtually no effect of the buoyancy model on the temperature fields. This outcome is to be expected for the models depicted in Figs. 1 and 2. As already noted, these models differ only by the inclusion or the omission of the pseudo-density term. The precisely identical temperature fields illustrated in Figs. 1 and 2 are further testimony to the assertion that the presence of the pseudo-density term does no harm when it is properly accounted.

The special feature of the density model of Fig. 3 is its isobaric nature. However, a comparison of Fig. 3 with either of Figs. 1 or 2 reveals only slight differences in detail. It may, therefore, be concluded that the accounting or non-accounting of the pressure variations is a lower-order effect.

Fig. 4 exhibits the temperature field that corresponds to the Boussinesq equation of state. A comparison of the isotherms of Fig. 4 with those of the preceding figures again reveals only slight differences. This outcome is further corroboration of the effectiveness of the reference temperature choice that was made in the evaluation of the thermophysical properties.

4.3. Velocity field comparisons

As a final demonstration of the effect of the buoyancy model, velocity field information is presented in

Table 1
Comparison of surface-averaged heat transfer coefficients for the six-sided thermal load as calculated from various models of the buoyancy term in the momentum equation

Surface	Ideal gas without pseudo-density difference	Ideal gas with pseudo-density difference	Isobaric ideal gas model with pseudo-density difference	Boussinesq model
	Per-surface- and all-surface-averaged heat transfer coefficients (W/m^2 K)			
$\bar{h}_{TOP}$	4.89	4.89	4.84	4.80
$\bar{h}_{BOT}$	5.87	5.87	5.96	6.00
$\bar{h}_{SIDE}$	7.27	7.27	7.33	7.26
$\bar{h}_{FRONT}$	6.81	6.79	6.85	6.90
$\bar{h}_{REAR}$	7.44	7.44	7.21	7.20
$\bar{\bar{h}}$	6.05	6.05	6.06	6.14

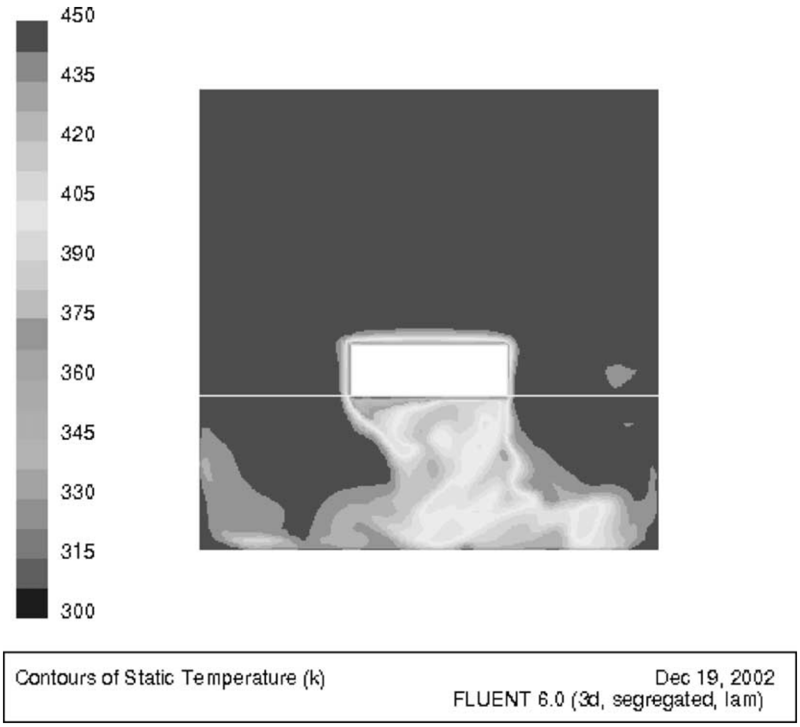

Fig. 1. Temperature field for ideal gas model without pseudo-density difference.

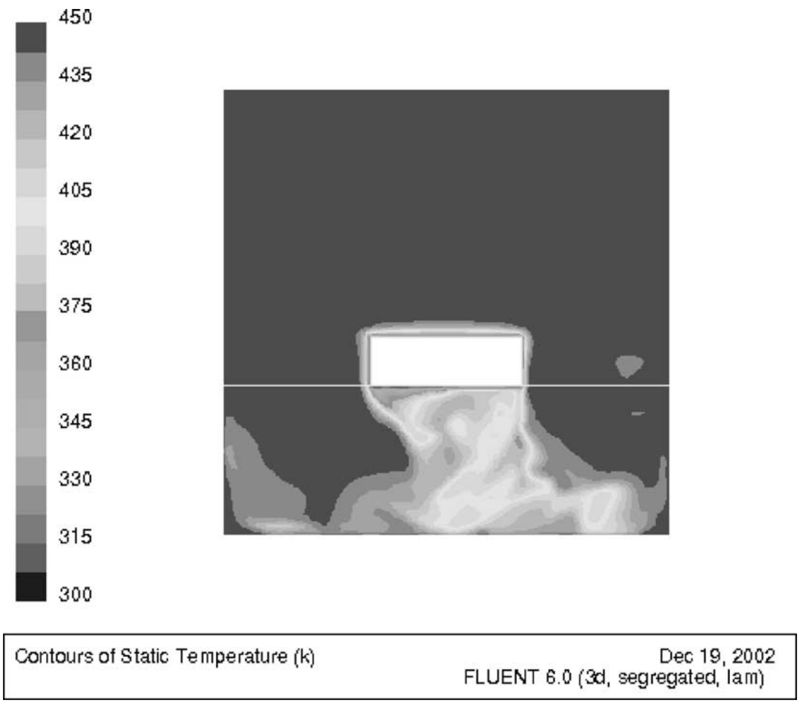

Fig. 2. Temperature field for ideal gas model with pseudo-density difference.

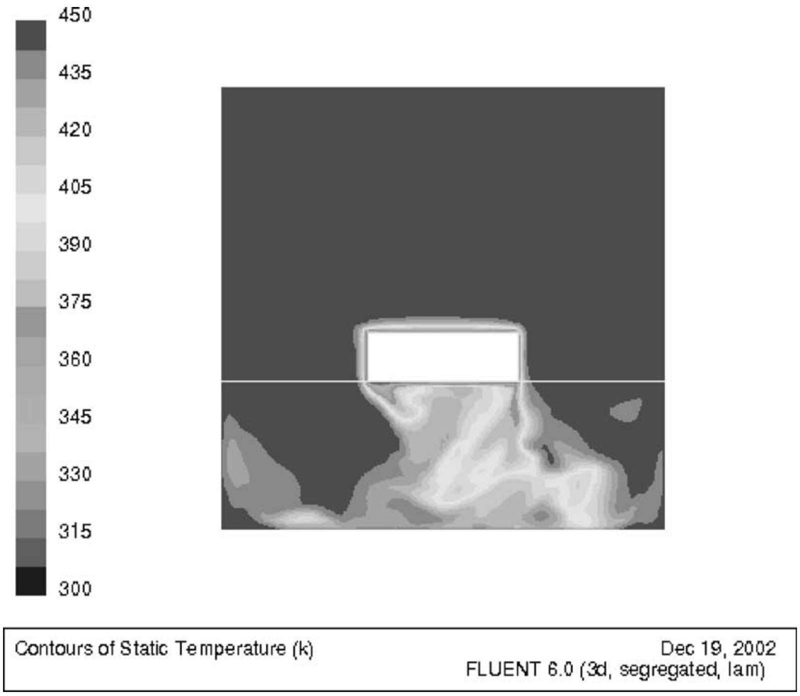

Fig. 3. Temperature field for isobaric ideal gas model with pseudo-density difference.

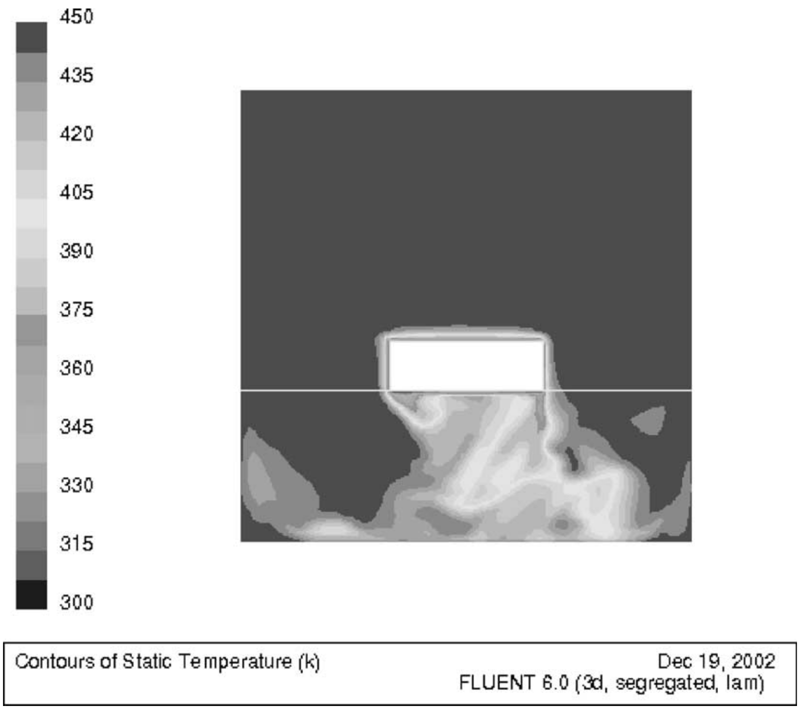

Fig. 4. Temperature field for Boussinesq model.

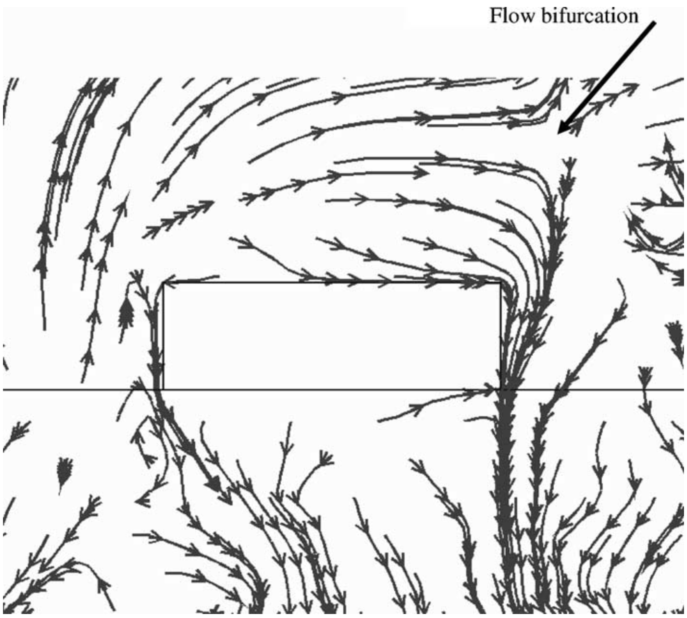

Fig. 5. Pathlines for the ideal gas model without pseudo buoyancy.

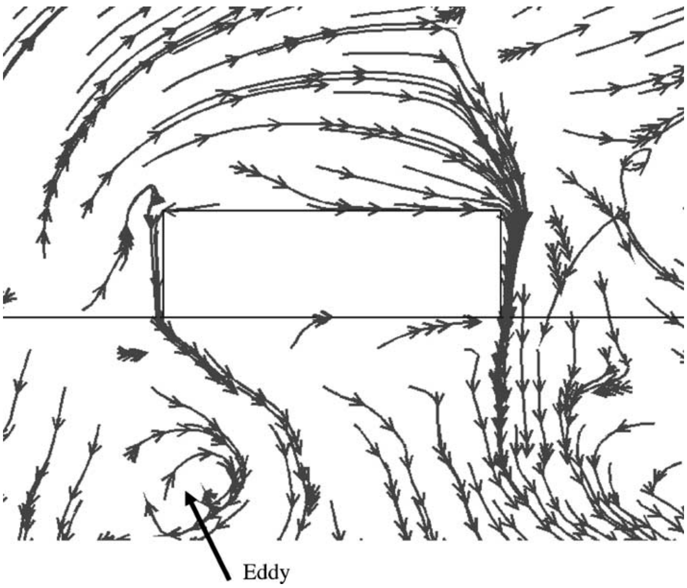

Fig. 6. Pathlines for the Boussinesq model.

Figs. 5 and 6. These figures, respectively, correspond to the ideal gas model without the pseudo-buoyancy term and to the Boussinesq model. The pathlines that are exhibited correspond to a symmetry plane that bisects the thermal load and the oven cavity. Careful inspection of the figures suggests an overall similarity in the pathlines, but there are interesting differences in detail. For example, the flow bifurcation that is identified in Fig. 5 occurs above the exhibited region in Fig. 6. On the other hand, the distinct eddy that is identified at the lower left of Fig. 6 has not quite formed in Fig. 5.

Although the aforementioned differences in the flow patterns are of theoretical interest, it is relevant to note that in the immediate neighborhood of the thermal load, the velocity fields for the two cases are very similar. It is believed that the near-wall velocity field is most significant in determining the surface heat transfer results.

5. Concluding remarks

It has been shown conclusively in this paper that the tradition of employing a pseudo-density difference to represent buoyancy for internal natural convection of ideal gases is irrelevant to the physical mechanisms that create the buoyancy force. When treated properly, the presence of a pseudo-density difference does no harm, but it tends to confuse the factors that create and sustain the natural convection. It was also demonstrated that the effect of pressure variations within the enclosed space has a negligible effect on the buoyancy and on the resulting surface heat transfer coefficients. In the absence of the ideal gas model, it was demonstrated that the Boussinesq equation of state gives satisfactory results provided that the density, thermal conductivity, viscosity, and coefficient of thermal expansion are all evaluated at a temperature that is the average of the temperatures of the walls which bound the enclosed space.

The validity of these conclusions and the accuracy of the numerically obtained heat transfer coefficients is strongly supported by experimental data collected by the authors. The agreement of the numerically determined and experimentally obtained heat transfer results was on the order of 1%.

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References

[1] E. Schmidt, W. Beckmann, Das Temperatur- und geschwindigkeitsfeld von einer warme abgebenden senkrechten platte bei naturlicher konvektion, Forsch. Ingenieurwes. 1 (1930) 391–397.

[2] G.d.V. Davis, Natural convection of air in a square cavity: a benchmark numerical solution, Int. J. Numer. Methods Fluids 3 (1983) 249–264.

[3] G.d.V. Davis, I.P. Jones, Natural convection in a square cavity—a comparison exercise, Int. J. Numer. Methods Fluids 3 (1983) 227–248.

[4] T.J. Heindel, S. Ramadhyani, F.P. Incropera, Laminar natural convection in a discretely heated cavity: I—assessment of three-dimensional effects, J. Heat Transfer 117 (1995) 902–909.

[5] D.C. Wan, B.S.V. Patnik, G.W. Wei, A new benchmark-quality solution for the buoyancy-driven cavity by discrete singular convolution, Numer. Heat Transfer Part B 40 (2001) 199–228.

[6] J.R. Fontaine, F. Biolley, R. Rap, J.C. Serieys, J.C. Cunin, Analysis of a 3D ventilation flow: experimental validation on a water scale model of numerical simulations, Numer. Heat Transfer Part A 26 (1994) 431–451.

[7] D.W. Pepper, K.G.T. Hollands, Summary of benchmark numerical studies for 3-D natural convection in an air-filled enclosure, Numer. Heat Transfer Part A 42 (2002) 1–11.

[8] R. Cheesewrite, K.J. Kung, S. Ziai, Experimental data for the validation of computer codes for the prediction of two-dimensional buoyant cavity flows, Proc. ASME Meeting, HTD 60 (1986) 75–81.

[9] W.H. Leong, K.G.T. Hollands, A.P. Brunger, Experimental Nusselt numbers for a cubical cavity benchmark problem in natural convection, Int. J. Heat Mass Transfer 42 (1999) 1979–1989.

[10] C.Y. Han, S.W. Baek, The effects of radiation on natural convection in a rectangular enclosure divided by two partitions, Numer. Heat Transfer, Part A 37 (2000) 249–270.

[11] K.S. Hung, C.H. Cheng, Pressure effects on natural convection for non-Boussinesq fluid in a rectangular enclosure, Numer. Heat Transfer, Part A 41 (2002) 515–528.

[12] Anon., Manual for FLUENT 6.0, FLUENT, Inc., Lebanon, NH.

[13] J.P. Abraham, E.M. Sparrow, Three-dimensional laminar and turbulent natural convection in a continuously/discretely wall-heated enclosure containing a thermal load, Numer. Heat Transfer, Part A 43 (2003) 1–21.